Int. J. Heat Mass Transfer. Vol. 34, No. 11, pp. 2859–2868, 1991
Printed in Great Britain

0017-9310/91 \$3.00+0.00
© 1991 Pergamon Press plc

Universal formulations and comprehensive correlations for non-Darcy natural convection and mixed convection in porous media

W. S. YU, H. T. LIN† and C. S. LU

Department of Chemical Engineering, National Central University, Chungli, Taiwan 32054, Republic of China

(*Received* 25 *July* 1990 *and in final form* 12 *December* 1990)

Abstract—Universal similarity equations of non-Darcy mixed convection along an isothermal vertical plate and a uniform-flux horizontal plate in porous media are obtained by introducing proper transformation variables. Very accurate similarity solutions and comprehensive correlation equations of the local Nusselt numbers are presented over the entire range of flow inertia including the Darcy and non-Darcy regimes for any mixed convection intensity from the pure forced convection limit to the pure natural convection limit. The finite-difference solution and correlation of the local Nusselt number for any extent of flow inertia are also presented for natural convection along a vertical plate with uniform wall heat flux.

1. INTRODUCTION

HEAT TRANSFER in fluid-saturated porous media has been studied extensively [1–15]. Although most of the studies were based on the Darcy flow model, non-Darcy convection has received more and more attention [6–15] since Vafai and Tien [6] first investigated the inertia and boundary effects on forced convection heat transfer. As has been shown in ref. [6] for forced convection and refs. [10–12] for natural convection, the inertia and boundary effects are very significant in high-porosity media but not very important in low-porosity media.

In the previous analyses [7–12] of non-Darcy natural convection along an isothermal vertical plate, different types of formulations have been used. Plumb and Huenefeld [7], and Chen and Ho [8], utilized the conventional similarity variables of Darcy natural convection [1] and introduced an inertial parameter Gr'. Bejan and Poulikakos [9] proposed similarity variables of proper scales for the non-Darcy flow limit, and used an inertial parameter G which is an inverse-square-root of Gr'.

In this paper, we propose new similarity variables that are novel combinations of the previous two types of similarity variables. A quite different inertial parameter is also introduced, which measures the whole range of flow inertia from 0 (Darcy flow) to 1 (non-Darcy flow limit for which inertia is completely dominant). With the present similarity variables and inertial parameter, the transformed similarity equations are valid uniformly over the Darcy and non-Darcy regimes of any flow inertia. In addition, a very accurate (maximum error less than 1.9%) correlation equation of the local Nusselt number for the entire range of flow inertia can be derived in terms of the inertial parameter and the dimensionless heat transfer groups defined for the cases of Darcy flow and non-Darcy flow limit. A similar solution method has been applied to the natural convection systems of a vertical plate and a horizontal plate maintained with uniform wall heat flux.

We begin the analysis from non-Darcy mixed convection to cover the cases of pure natural convection and pure forced convection. For non-Darcy mixed convection from an isothermal vertical plate and a horizontal plate with uniform heat flux, universal similarity equations for the entire regimes of flow inertia and mixed convection intensity can be obtained by introducing a proper mixed convection parameter and dimensionless transformation variables. The present similarity equations differ markedly from the previously reported ones [4, 5, 13–15]. In terms of the properly defined parameters of inertia and buoyancy, we have derived comprehensive correlation equations of the local Nusselt numbers for any mixed convection intensity from the pure forced convection limit to the pure natural convection limit over the entire range of flow inertia. The maximum deviation of the correlation from the numerical results is less than 6% in the whole domain of flow inertia and mixed convection intensity. To the knowledge of the authors, the correlation equation of non-Darcy natural convection or mixed convection in porous media does not seem to have been reported previously.

2. MATHEMATICAL FORMULATION

2.1. *System equations*

Consider the mixed convection flow along a vertical or a horizontal flat plate embedded in a fluid-saturated

† Author to whom correspondence should be addressed.

NOMENCLATURE

- A dimensionless heat transfer groups of Darcy natural convection
- b inertia coefficient in the Forchheimer model [m^{-1}]
- B dimensionless natural convection heat transfer groups of non-Darcy flow limit
- C dimensionless heat transfer group of forced convection, $Nu_F/Pe^{1/2}$
- f dimensionless stream function
- g gravitational acceleration [$m\,s^{-2}$]
- h local heat transfer coefficient [$W\,m^{-2}\,K^{-1}$]
- k thermal conductivity of fluid-saturated porous medium [$W\,m^{-1}\,K^{-1}$]
- K permeability [m^2]
- m constant exponent
- n constant exponent
- Nu local Nusselt number
- p pressure [$N\,m^{-2}$]
- Pe local Peclet number, $u_\infty x/\alpha$
- q heat flux [$W\,m^{-2}$]
- R buoyancy parameter defined in equations (12) and (13)
- Ra_D Darcy-modified Rayleigh number for the UWT case, $g\beta(T_w-T_\infty)Kx/\alpha\nu$
- Ra_D^* Darcy-modified Rayleigh number for the UHF case, $g\beta(q_w x/k)Kx/\alpha\nu$
- Ra_n modified Rayleigh number for the non-Darcy flow limit, UWT case, $g\beta(T_w-T_\infty)(x^2/b)/\alpha^2$
- Ra_n^* modified Rayleigh number for the non-Darcy flow limit, UHF case, $g\beta(q_w x/k)(x^2/b)/\alpha^2$
- T temperature [K]
- u, v volume-averaged velocity components in the x- and y-direction [$m\,s^{-1}$]
- x, y coordinates parallel and normal to the plate [m].

Greek symbols

- α effective thermal diffusivity of the saturated porous medium [$m^2\,s^{-1}$]
- β coefficient of thermal expansion [K^{-1}]
- ζ inertial parameter defined in equations (14) and (15)
- λ $Pe^{1/2}+R$
- η dimensionless coordinate, $(y/x)\lambda$
- θ dimensionless temperature: $(T-T_\infty)/(T_w-T_\infty)$ for the UWT case; $(T-T_\infty)\lambda/(q_w x/k)$ for the UHF case
- ν kinematic viscosity of the fluid [$m^2\,s^{-1}$]
- ξ mixed convection parameter, $(1+Pe^{1/2}/R)^{-1}$
- ρ fluid density [$kg\,m^{-3}$]
- ϕ angle of plate inclination measured from the horizontal
- ψ stream function.

Subscripts

- D Darcy
- F forced convection
- M mixed convection
- n non-Darcy
- N natural convection
- w at the wall
- ∞ in the ambient.

porous medium which is homogeneous, isotropic and in thermal equilibrium. The plate is maintained either with a uniform wall temperature (UWT) or a uniform wall heat flux (UHF). Physical properties such as viscosity, heat conductivity and thermal expansion coefficient are assumed to be constant. In addition, we assume the Boussinesq and boundary-layer approximations are valid. Under the above assumptions and using the flow model of Forschheimer, the volume-averaged conservation equations can be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u(\nu/K+bu) = -\frac{1}{\rho}\frac{\partial p}{\partial x} \pm g\beta(T-T_\infty)\sin\phi \tag{2}$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial y}+g\beta(T-T_\infty)\cos\phi \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{4}$$

where K and α are the permeability and the equivalent thermal diffusivity of the fluid-saturated porous medium, and b is an inertial coefficient in the Forchheimer model. The inclination angle ϕ is measured from the horizontal ($\phi = 0$) to the vertical ($\phi = \pi/2$). The plus sign in front of the last term of equation (2) denotes natural convection and buoyancy-assisted mixed convection, while the minus sign denotes buoyancy-opposed mixed convection.

The appropriate boundary conditions are

$$\text{at} \quad y=0,\quad v=0,\quad T=T_w \quad \text{or} \quad -k(\partial T/\partial y)=q_w \tag{5}$$

$$\text{as} \quad y\to\infty,\quad u=u_\infty,\quad T=T_\infty. \tag{6}$$

For natural convection, $u_\infty = 0$.

2.2. *New transformation variables*

A new dimensionless coordinate for analysing mixed convection from a vertical or a horizontal flat plate in porous media is defined as

$$\eta = (y/x)\lambda \tag{7}$$

with

$$\lambda = Pe^{1/2} + R \tag{8}$$

where the Peclet number Pe is defined as

$$Pe = u_\infty x/\alpha \tag{9}$$

and the variable R that represents the buoyancy effect is an appropriate combination of the Darcy-modified Rayleigh number

$$Ra_D = g\beta(T_w - T_\infty)Kx/\alpha\nu \quad \text{UWT} \tag{10a}$$

$$Ra_D^* = g\beta(q_w x/k)Kx/\alpha\nu \quad \text{UHF} \tag{10b}$$

and the modified Rayleigh number for the non-Darcy flow limit

$$Ra_n = g\beta(T_w - T_\infty)(x^2/b)/\alpha^2 \quad \text{UWT} \tag{11a}$$

$$Ra_n^* = g\beta(q_w x/k)(x^2/b)/\alpha^2 \quad \text{UHF}. \tag{11b}$$

For natural convection along a vertical plate in porous media, R is defined as

$$R = (Ra_D^{-1/2} + Ra_n^{-1/4})^{-1} \quad \text{UWT} \tag{12a}$$

$$R = (Ra_D^{*-1/3} + Ra_n^{*-1/5})^{-1} \quad \text{UHF} \tag{12b}$$

while for natural convection over a horizontal plate

$$R = (Ra_D^{-1/3} + Ra_n^{-1/5})^{-1} \quad \text{UWT} \tag{13a}$$

$$R = (Ra_D^{*-1/4} + Ra_n^{*-1/6})^{-1} \quad \text{UHF}. \tag{13b}$$

We also introduce new inertial parameters as

$$\zeta = (1 + Ra_n^{1/4}/Ra_D^{1/2})^{-1} \quad \text{UWT} \tag{14a}$$

$$\zeta = (1 + Ra_n^{*1/5}/Ra_D^{*1/3})^{-1} \quad \text{UHF} \tag{14b}$$

for the cases of a vertical plate; and

$$\zeta = (1 + Ra_n^{1/5}/Ra_D^{1/3})^{-1} \quad \text{UWT} \tag{15a}$$

$$\zeta = (1 + Ra_n^{*1/6}/Ra_D^{*1/4})^{-1} \quad \text{UHF} \tag{15b}$$

for the cases of a horizontal plate. The inertial parameter for each case describes the strength of inertial effects. For the limiting case of Darcy flow with inertia totally neglected, $\zeta = 0$. Whereas $\zeta = 1$ represents the case of non-Darcy flow limit for which inertia is completely dominant.

A new mixed convection parameter is proposed here as

$$\xi = (1 + Pe^{1/2}/R)^{-1} \tag{16}$$

where the buoyancy effect parameter R has been defined in equations (12a), (12b) and (13a), (13b) for various natural convection cases. The mixed convection parameter ξ is a measure of the relative intensity of natural convection to forced convection. For the case of pure forced convection, $\xi = 0$. Whereas for the case of pure natural convection, $\xi = 1$.

In addition to the dimensionless coordinates and parameter, we also defined the dimensionless stream function

$$f = \psi/\alpha\lambda \tag{17}$$

and the dimensionless temperature

$$\theta = (T - T_\infty)/(T_w - T_\infty) \quad \text{UWT} \tag{18a}$$

$$\theta = (T - T_\infty)\lambda/(q_w x/k) \quad \text{UHF}. \tag{18b}$$

2.3. *Similarity equations of a vertical plate with UWT*

For mixed convection along a vertical plate with uniform wall temperature (UWT), equations (1)–(6) can be transformed into the following similarity equations:

$$2\zeta^4 f'f'' + (1-\zeta)^2\xi^2 f'' = \pm\xi^4\theta' \tag{19}$$

$$2\theta'' + f\theta' = 0 \tag{20}$$

$$f(0) = 0, \quad \theta(0) = 1 \tag{21a,b}$$

$$f'(\infty) = (1-\xi)^2, \quad \theta(\infty) = 0. \tag{22a,b}$$

The plus and minus signs in front of the right-hand side of equation (19) represent the buoyancy-assisting and buoyancy-opposing mixed convection, respectively. The local Nusselt number of this mixed convection system can be expressed as

$$Nu_M = hx/k = \lambda[-\theta'(0)] \tag{23a}$$

$$= \xi^{-1}(1-\zeta)Ra_D^{1/2}[-\theta'(0)] \tag{23b}$$

$$= \xi^{-1}\zeta\, Ra_n^{1/4}[-\theta'(0)] \tag{23c}$$

$$= (1-\xi)^{-1}\, Pe^{1/2}[-\theta'(0)]. \tag{23d}$$

The transformed similarity equations for pure natural convection along an isothermal vertical plate are readily obtained from equations (19)–(22) by letting $\xi = 1$. The reduced momentum equation is

$$2\zeta^4 f'f'' + (1-\zeta)^2 f'' - \theta' = 0 \tag{24}$$

with the boundary condition $f'(\infty) = 0$ reduced from equation (22a). The local Nusselt number reduced from equation (23) is

$$Nu_N = (1-\zeta)Ra_D^{1/2}[-\theta'(0)] \tag{25a}$$

$$= \zeta\, Ra_n^{1/4}[-\theta'(0)]. \tag{25b}$$

On the other hand, the transformed similarity equations and the local Nusselt number for pure forced convection are obtained from equations (19) to (23) by letting $\xi = 0$. The local Nusselt number for forced convection is obtained as

$$Nu_F = Pe^{1/2}[-\theta'(0)]. \tag{26}$$

2.4. *Similarity equations of a horizontal plate with UHF*

For mixed convection along a horizontal plate with uniform wall heat flux (UHF), the transformed similarity equations are

$$4\zeta^6 f'f'' + 2(1-\zeta)^4\xi^2 f'' = \pm\xi^6(\eta\theta' - \theta) \tag{27}$$

$$2\theta'' + f\theta' - f'\theta = 0 \tag{28}$$

$$f(0) = 0, \quad \theta'(0) = -1 \tag{29}$$

$$f'(\infty) = (1-\xi)^2, \quad \theta(\infty) = 0. \tag{30}$$

The local Nusselt number is expressed as

$$Nu_M = hx/k = \lambda[1/\theta(0)] \tag{31a}$$

$$= \xi^{-1}(1-\zeta)Ra_D^{*1/4}[1/\theta(0)] \tag{31b}$$

$$= \xi^{-1}\zeta\, Ra_n^{*1/6}[1/\theta(0)] \tag{31c}$$

$$= (1-\xi)^{-1}Pe^{1/2}[1/\theta(0)]. \tag{31d}$$

For natural convection along a constant-flux horizontal plate, the similarity momentum equation reduced from equation (27) by letting $\xi = 1$ is obtained as

$$4\zeta^6 f'f'' + 2(1-\zeta)^4 f'' - \eta\theta' + \theta = 0. \tag{32}$$

The local Nusselt number is

$$Nu_N = (1-\zeta)Ra_D^{*1/4}[1/\theta(0)] \tag{33a}$$

$$= \zeta\, Ra_n^{*1/6}[1/\theta(0)]. \tag{33b}$$

2.5. *Non-similarity equations of non-Darcy natural convection along a vertical plate with UHF*

For non-Darcy natural convection along a vertical plate with uniform wall heat flux (UHF), the inertial parameter defined in equation (14b) is dependent on x. Therefore, this system will not permit similarity solutions. The transformed non-similarity equations and boundary conditions are

$$2\zeta^5 f'f'' + (1-\zeta)^3 f'' - \theta' = 0 \tag{34}$$

$$\theta'' + \frac{10-\zeta}{15} f\theta' - \frac{5+\zeta}{15} f'\theta = \tfrac{1}{15}\zeta(1-\zeta)\left[f'\frac{\partial\theta}{\partial\zeta} - \theta'\frac{\partial f}{\partial\zeta}\right] \tag{35}$$

$$f(\zeta, 0) = 0, \quad \theta'(\zeta, 0) = -1 \tag{36}$$

$$f'(\zeta, \infty) = 0, \quad \theta(\zeta, \infty) = 0. \tag{37}$$

The local Nusselt number for this system is

$$Nu_N = (1-\zeta)Ra_D^{*1/3}[1/\theta(\zeta, 0)] \tag{38a}$$

$$= \zeta\, Ra_n^{*1/5}[1/\theta(\zeta, 0)]. \tag{38b}$$

For Darcy ($\zeta = 0$) and extremely non-Darcy ($\zeta = 1$) natural convection, equation (35) can be reduced to the similarity equation for each case.

3. CORRELATIONS

3.1. *Natural convection*

For natural convection in porous media over the entire range of flow inertia, we propose a correlation equation of the form

$$(1/Nu_N)^m = (1/Nu_D)^m + (1/Nu_n)^m, \quad m > 0 \tag{39}$$

where Nu_D and Nu_n are the Nusselt numbers of natural convection for the asymptotic cases of Darcy and non-Darcy flow limits, respectively. The exponent m is a positive constant that can be specified by comparison with the numerical data. Equation (39) can be written alternatively as

$$Nu_N/R = [A^{-m}(1-\zeta)^{-m} + B^{-m}\zeta^{-m}]^{-1/m} \tag{40}$$

where the dimensionless natural convection heat transfer groups A and B, respectively for the Darcy flow and the non-Darcy flow limit, are defined in Table 1 for the various cases of natural convection.

3.2. *Mixed convection*

The basic form of the correlation equation of mixed convection in porous media is essentially the same as that in a single phase fluid introduced by Churchill [16]

$$Nu_M^n = Nu_F^n \pm Nu_N^n, \quad n > 0 \tag{41}$$

where Nu_F and Nu_N are the local Nusselt numbers of forced and natural convection, respectively. The plus and minus signs in front of the last term of equation (41) refer to the buoyancy-assisting and buoyancy-opposing cases, respectively. By substituting equation (39) into equation (41), we obtain

$$Nu_M^n = Nu_F^n \pm [(Nu_D^{-m} + Nu_n^{-m})^{-1/m}]^n. \tag{42}$$

This equation can be expressed alternatively as

$$Nu_M/\lambda = \{C^n(1-\xi)^n \pm [A^{-m}(1-\zeta)^{-m} + B^{-m}\zeta^{-m}]^{-n/m}\xi^n\}^{1/n} \tag{43}$$

where the dimensionless heat transfer groups A and B for various natural convection cases are presented in Table 1. The dimensionless forced convection heat transfer group C is defined as $C = Nu_F/Pe^{1/2}$.

Equation (43) can also be rewritten as

$$Y^n = 1 \pm X^n \tag{44}$$

where

$$Y = \frac{Nu_M/\lambda}{C(1-\xi)} \tag{45}$$

and

$$X = \frac{\xi[A^{-m}(1-\zeta)^{-m} + B^{-m}\zeta^{-m}]^{-1/m}}{C(1-\xi)}. \tag{46}$$

4. RESULTS AND DISCUSSION

4.1. *Numerical results*

Precise numerical solutions for the similarity equations of mixed convection or natural convection can be obtained easily by using the shooting method and a fourth-order Runge–Kutta integration scheme. They were also solved by Keller's Box method [17] to check the accuracy. The non-similarity partial differential equations (34)–(37) for the case of natural convection along a vertical plate with UHF are solved by Keller's Box method. To conserve space, only the numerical results of the dimensionless heat transfer groups A, B, and C in correlation equations (40) and (43) are presented. The present results of A and B for the

Table 1. The definitions and values of the dimensionless heat transfer groups A and B for various natural convection cases

Cases	A for Darcy flow	B for non-Darcy flow limit
Vertical plate		
UWT	$Nu_D/Ra_D^{1/2} = 0.44388$	$Nu_n/Ra_n^{1/4} = 0.49380$
	0.4440 [1]	0.494 [9]
	0.44390 [7]	
UHF	$Nu_D/Ra_D^{*1/3} = 0.77149$	$Nu_n/Ra_n^{*1/5} = 0.80573$
	0.7723 [3]	0.804 [9]
Horizontal plate		
UHF	$Nu_D/Ra_D^{*1/4} = 0.85884$	$Nu_n/Ra_n^{*1/6} = 0.85098$
	0.8588 [3]	

various natural convection cases of Darcy flow and non-Darcy flow limit are listed and compared with the reported data [1, 3, 7, 9] in Table 1. The value of $C = Nu_F/Pe^{1/2}$ for pure forced convection of the UWT case is 0.56423 and that of the UHF case is 0.88616. They coincide excellently with the reported data of 0.5641 [4] and 0.8862 [5], respectively.

4.2. *Natural convection*

Typical dimensionless longitudinal velocity profiles $f'(\eta) = u/(\alpha/x)R^2$ are presented in Fig. 1 for the case of a vertical plate with UWT. While typical profiles of the dimensionless temperature are shown in Figs. 2 and 3 for vertical plates with UWT and UHF, respectively. As can be seen from these figures, the profiles for various flow inertia are very similar. This indicates that the present definitions of the similarity variables and the inertial parameters are very appropriate.

The decrease of $Nu_N/Ra_D^{*1/3}$ with increasing the inertial parameter $Ra_D^{*1/3}/Ra_n^{*1/5} = \zeta/(1-\zeta)$ is shown in Fig. 4 for the case of a vertical plate with UHF.

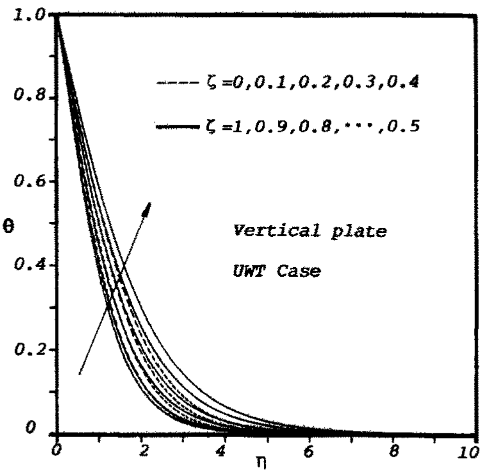

FIG. 2. Dimensionless temperature profiles of natural convection along a vertical plate with UWT.

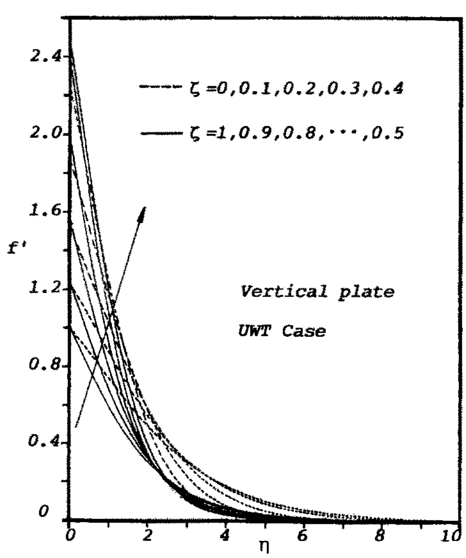

FIG. 1. Dimensionless velocity profiles of natural convection along a vertical plate with UWT.

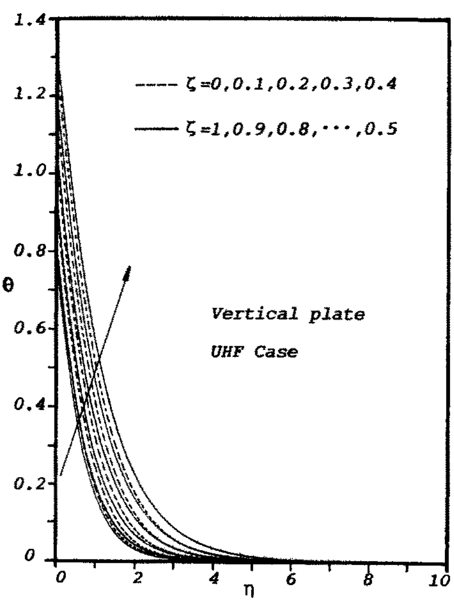

FIG. 3. Dimensionless temperature profiles of natural convection along a vertical plate with UHF.

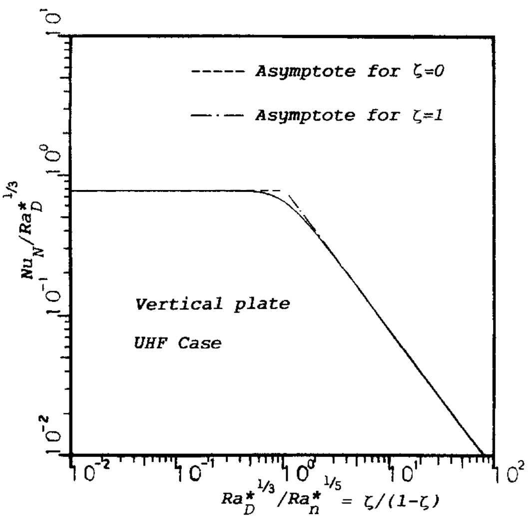

FIG. 4. Heat transfer results for non-Darcy natural convection along a vertical plate with UWT.

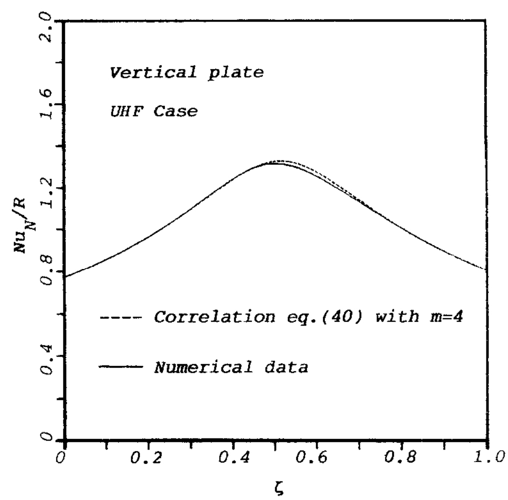

FIG. 6. A comparison between the correlated and the calculated Nu_N for non-Darcy natural convection along a vertical plate with UHF.

This figure clearly shows an asymptotic regime of Darcy flow, an intermediate regime about $Ra_D^{*\,1/3}/Ra_n^{*\,1/5} = 1$ or $\zeta = 0.5$, and an asymptotic regime of non-Darcy flow limit. The corresponding curve for a plate with UWT will be shown in Fig. 12 as a special case ($\xi = 1$).

For natural convection along a vertical plate, the correlation equation (40) with $m = 3$ for the UWT case and that with $m = 4$ for the UHF case are presented in Figs. 5 and 6, respectively. Comparisons between the correlated results and the numerical data show that the agreement is excellent. The maximum deviation between the correlated and calculated local Nusselt number over the entire regime of flow inertia is less than 2.99% for the UWT case and is less than 1.7% for the UHF case. If we set $m = 2.8$ instead of 3 for the UWT case, the maximum error will be reduced from 2.99 to 1.9%. For the case of a horizontal plate with UHF, the maximum error of the correlation equation (40) with $m = 5$ is less than 0.91% over the whole inertia regime.

4.3. *Mixed convection*

Representative profiles of the dimensionless velocity $f'(\eta) = u/(\alpha/x)\lambda^2$ and the dimensionless temperature $\theta(\eta)$ are respectively shown in Figs. 7 and 8 for the case of a vertical plate with UWT. These figures show the evolution of the profiles from the forced convection limit ($\xi = 0$) to the natural convection

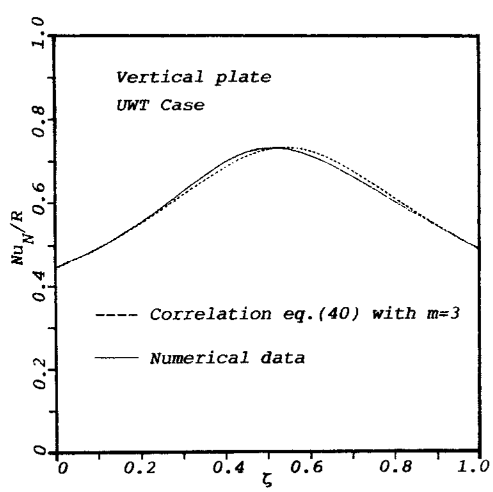

FIG. 5. A comparison between the correlated and the calculated Nu_N for non-Darcy natural convection along a vertical plate with UWT.

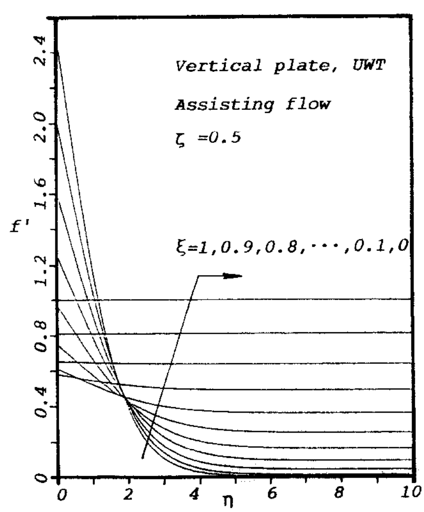

FIG. 7. Variations of the dimensionless velocity profiles with ξ for mixed convection along a vertical plate with UWT, $\zeta = 0.5$.

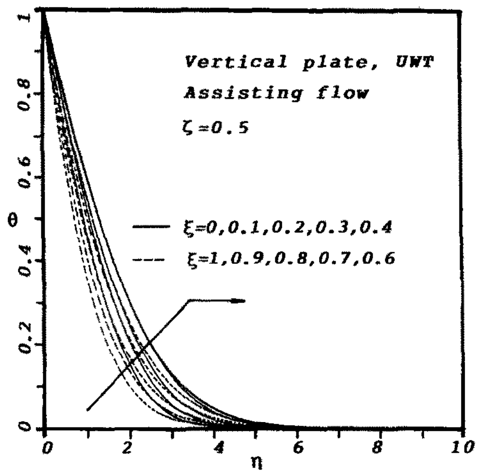

FIG. 8. Variations of the dimensionless temperature profiles with ξ for mixed convection along a vertical plate with UWT, $\zeta = 0.5$.

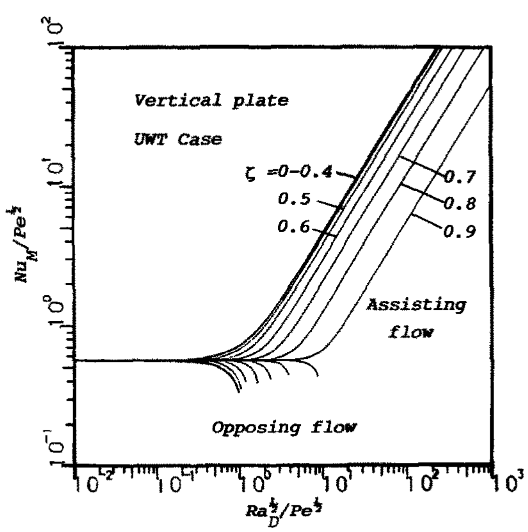

FIG. 10. Variations of $Nu_M/Pe^{1/2}$ with $Ra_D^{1/2}/Pe^{1/2}$ for mixed convection along a vertical plate with UWT.

limit ($\xi = 1$). We plot these figures with $\zeta = 0.5$ since the intermediate regime is around the value of the inertial parameter, as can be seen from Figs. 4–6.

The effect of flow inertia to the dimensionless temperature profile of a vertical plate with UWT is presented in Fig. 9 for a typical assisting mixed convection flow of $\xi = 0.5$. The increase of flow inertia resulted in an increase of boundary-layer thickness and consequently caused a decrease of heat transfer rate, as will be seen in Figs. 10 and 12.

The effects of flow inertia and mixed convection intensity on the local heat transfer rate for a vertical plate with UWT are shown in Fig. 10. While their effects on the local wall temperature for a horizontal plate with UHF are shown in Fig. 11. Each of these figures shows that the local Nusselt number increases from the forced convection asymptote to the natural convection asymptotes of different ζ as the buoyancy parameter $Ra_D^{1/2}/Pe^{1/2}$ or $Ra_D^{*1/4}/Pe^{1/2}$ increases. With regard to the effect of flow inertia, Figs. 10 and 11 show that, for the assisting mixed convection, the local Nusselt number decreases as the inertial parameter ζ increases. Whereas for the opposing mixed convection, the effect is reversed. The decrease of the local Nusselt numbers with increasing flow inertia can also be seen in Figs. 12 and 13. For specific mixed convection intensities ($\xi = 0.1$–1), the local Nusselt numbers decrease from the Darcy asymptotes through an intermediate region to the asymptotes of the non-Darcy flow limit.

Comparisons between the correlated results and the

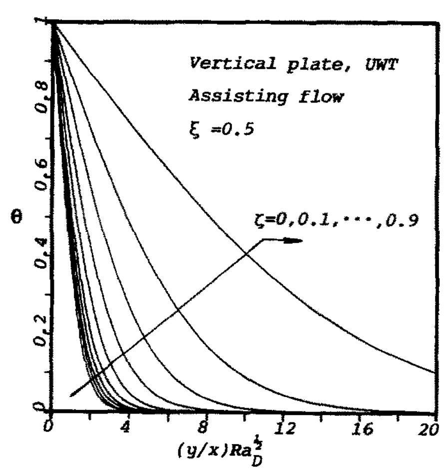

FIG. 9. Variations of the dimensionless temperature profiles with ζ for mixed convection along a vertical plate with UWT, $\xi = 0.5$.

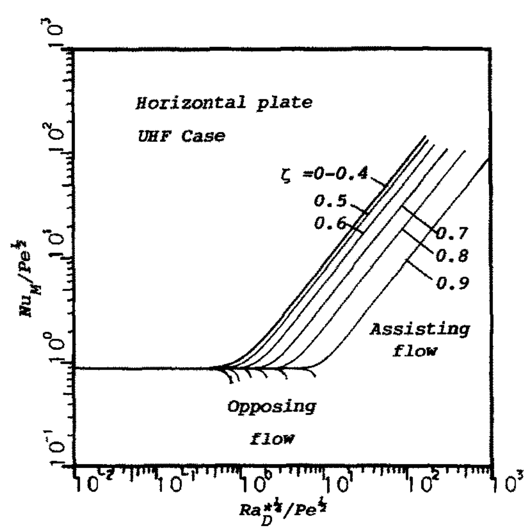

FIG. 11. Variations of $Nu_M/Pe^{1/2}$ with $Ra_D^{*1/4}/Pe^{1/2}$ for mixed convection over a horizontal plate with UHF.

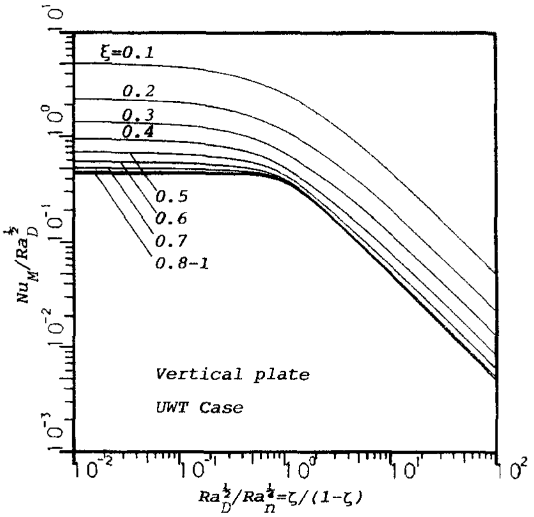

FIG. 12. Variations of $Nu_M/Ra_D^{1/2}$ with $Ra_D^{1/2}/Ra_n^{1/4}$ for mixed convection along a vertical plate with UWT.

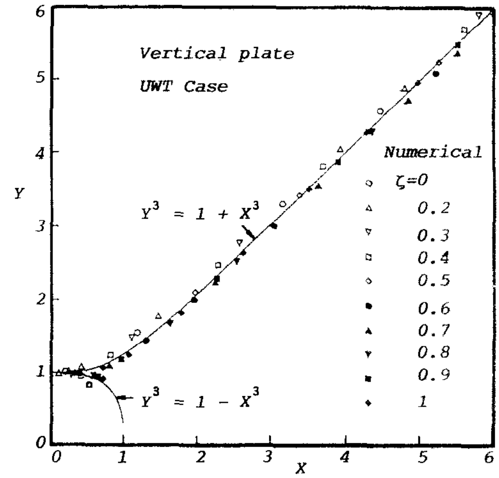

FIG. 14. A comparison between the correlated and calculated Nu_M for mixed convection along a vertical plate with UWT.

numerical data of mixed convection are presented in Figs. 14 and 15 for the cases of a vertical plate with UWT and a horizontal plate with UHF, respectively. Over the entire regimes of flow inertia ($0 \leqslant \zeta \leqslant 1$) and mixed convection intensity ($0 \leqslant \xi \leqslant 1$ for assisting flow, and $0 \leqslant \xi < 0.3$ for opposing flow before boundary-layer separation occurs), the maximum error of equation (43) with $m = n = 3$ is 11.3% for assisting flow, and 8.25% for opposing flow, for the case of a vertical plate with UWT. While for a horizontal plate with UHF, the maximum error of equation (43) with $m = n = 4$ is 5.87% for assisting flow and 4.05% for opposing flow. The maximum error of the correlation equation can be reduced significantly if different pairs of m and n are taken for different mixed convection intensity over the whole range of flow inertia, or for different flow inertia over the entire mixed convection regime. The maximum error of the correlation for a vertical plate with UWT can thus be reduced from 11.3% (when $m = n = 3$ is taken for any inertia) to less than 6%, as shown in Table 2.

For Darcy mixed convection ($\zeta = 0$), the maximum error of equation (43) with $n = 2$ is within 0.44% for assisting flow over the whole mixed convection regime, and 0.2% for opposing flow. Figure 16 shows the excellent agreement between the correlated and the calculated local Nusselt numbers for Darcy mixed convection along a vertical plate with UWT.

5. CONCLUSIONS

We have introduced appropriate parameters of inertia and mixed convection, and transformation

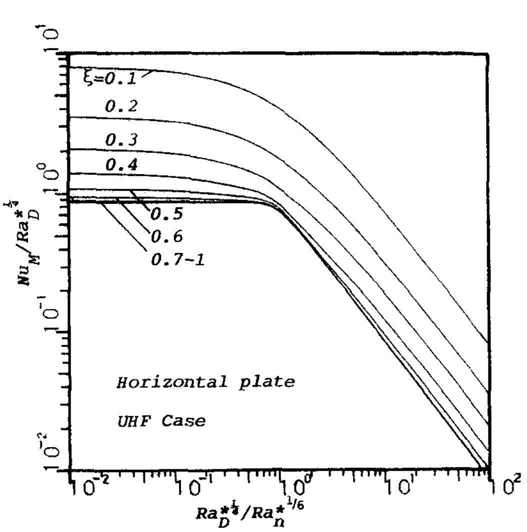

FIG. 13. Variations of $Nu_M/Ra_D^{*1/4}$ with $Ra_D^{*1/4}/Ra_n^{*1/6}$ for mixed convection over a horizontal plate with UHF.

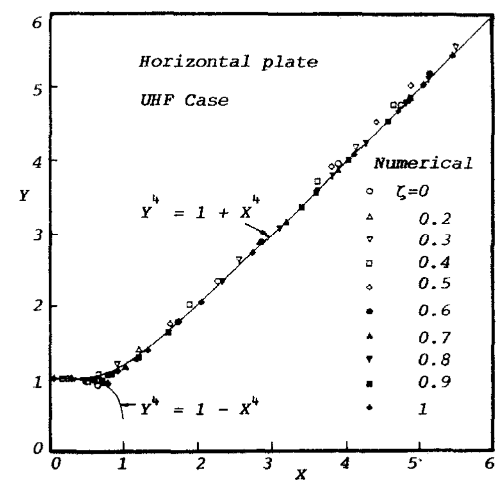

FIG. 15. A comparison between the correlated and calculated Nu_M for mixed convection over a horizontal plate with UHF.

Table 2. Values of m and n and the maximum error of the correlations over the whole range of mixed convection

ζ	m	n	Maximum error (%) Assisting	Maximum error (%) Opposing
0		2	0.44	0.20
0.1	2	2	0.73	0.33
0.2	2	2	2.27	0.61
0.3	2	2	5.55	0.32
0.4	2	3	5.92	4.38
0.5	3	3	3.02	0.67
0.6	3	3	2.86	2.28
0.7	3	3	4.75	2.67
0.8	4	4	1.93	0.11
0.9	4	4	0.57	0.07
1.0		4	0.68	0.05

variables that are within proper scales for any flow inertia and arbitrary mixed convection intensity. For non-Darcy mixed convection from a vertical plate with UWT and a horizontal plate with UHF, universal similarity equations have been derived, which are readily reducible to the conventional equations of various special cases. Simple but comprehensive and very accurate correlations of the local Nusselt numbers have been developed in terms of the inertial parameter, the mixed convection parameter, and the dimensionless heat transfer groups of forced convection and natural convection. The deviations between the numerical data and the predicted results from the correlation equations are within 2% for non-Darcy natural convection of any flow inertia, and 6% for mixed convection over the entire regimes of inertia and buoyancy.

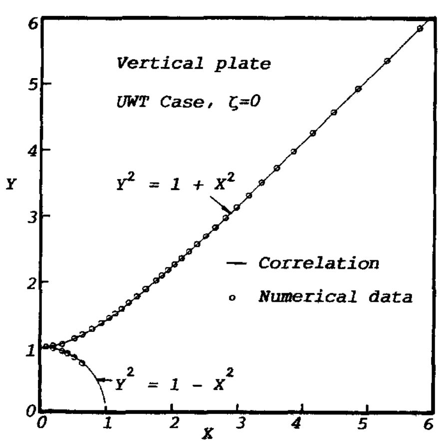

FIG. 16. A comparison between the correlated and calculated Nu_M for Darcy mixed convection along a vertical plate with UWT.

The present correlation and method of analysis can be applied to the other natural convection and mixed convection systems in porous media. The systems with thermal dispersion effects taken into account can also be analyzed by a similar procedure.

Acknowledgement—The support of the National Science Council of R.O.C. for this work is gratefully acknowledged.

REFERENCES

1. P. Cheng and W. J. Minkowycz, Free convection about a vertical flat plate imbedded in a porous medium with application to heat transfer from a dike, *J. Geophys. Res.* **82,** 2040–2044 (1977).
2. P. Cheng and I.-Dee Chang, Buoyancy induced flow in a saturated porous medium adjacent to impermeable horizontal surface, *Int. J. Heat Mass Transfer* **19,** 1267–1272 (1976).
3. P. Cheng, Constant surface heat flux solutions for porous layer flows, *Lett. Heat Mass Transfer* **4,** 119–128 (1977).
4. P. Cheng, Combined free and forced convection flow about inclined surfaces in porous media, *Int. J. Heat Mass Transfer* **20,** 807–814 (1977).
5. P. Cheng, Similarity solutions for mixed convection from horizontal impermeable surfaces in saturated porous media, *Int. J. Heat Mass Transfer* **20,** 893–898 (1977).
6. K. Vafai and C. L. Tien, Boundary and inertia effects on flow and heat transfer in porous media, *Int. J. Heat Mass Transfer* **24,** 195–204 (1981).
7. O. A. Plumb and J. C. Huenefeld, Non-Darcy natural convection from heated surface in saturated porous media, *Int. J. Heat Mass Transfer* **24,** 765–768 (1981).
8. K. S. Chen and J. R. Ho, Effects of flow inertia on vertical, natural convection in saturated, porous media, *Int. J. Heat Mass Transfer* **29,** 753–759 (1986).
9. A. Bejan and D. Poulikakos, The non-Darcy regime for vertical boundary layer natural convection in a porous medium, *Int. J. Heat Mass Transfer* **27,** 717–722 (1984).
10. J. T. Hong, C. L. Tien and M. Kaviany, Non-Darcian effects on vertical-plate natural convection in porous media with high porosities, *Int. J. Heat Mass Transfer* **28,** 2149–2157 (1985).
11. J. T. Hong, Y. Yamada and C. L. Tien, Effects of non-Darcian and nonuniform porosity on vertical-plate natural convection in porous media, *J. Heat Transfer* **109,** 356–362 (1987).
12. M. Kaviany and M. Mittal, Natural convection heat transfer from a vertical plate to high permeability porous media : an experiment and an approximate solution, *Int. J. Heat Mass Transfer* **30,** 967–977 (1987).
13. P. Ranganathan and R. Viskanta, Mixed convection boundary-layer flow along a vertical surface in a porous medium, *Numer. Heat Transfer* **7,** 305–317 (1984).
14. F. C. Lai and F. A. Kulacki, Non-Darcy convection from horizontal impermeable surfaces in saturated porous media, *Int. J. Heat Mass Transfer* **30,** 2189–2192 (1987).
15. F. C. Lai and F. A. Kulacki, Effects of flow inertia on mixed convection along a vertical surface in a saturated porous media, *ASME Proc. 1988 Natn. Heat Transfer Conf.*, Houston, Texas, Vol. 1, pp. 643–652 (1988).
16. S. W. Churchill, A comprehensive correlating equation for laminar, assisting, forced and free convection, *A.I.Ch.E. Jl* **23,** 10–16 (1977).
17. H. B. Keller, Numerical methods in boundary-layer theory, *A. Rev. Fluid Mech.* **10,** 417–433 (1978).

FORMULATIONS UNIVERSELLES ET CORRELATIONS ETENDUES POUR LA CONVECTION NATURELLE NON-DARCYENNE ET CONVECTION MIXTE DANS DES MILIEUX POREUX

Résumé—Des équations universelles de similitude pour la convection mixte non-darcyenne le long d'une plaque verticale isotherme et une plaque horizontale dans des milieux poreux sont obtenues en introduisant des variables convenables de transformation. Des solutions affines trés précises et des corrélations étendues de nombres de Nusselt locaux sont présentées dans le domaine entier d'écoulement incluant les régimes darcyen et non-darcyen pour une convection mixte quelconque entre la limite de convection forcée pure et celle de la convection naturelle pure. La solution aux différences finies et la corrélation du nombre de Nusselt local sont aussi présentées pour la convection naturelle le long d'une plaque verticale avec densité de flux thermique uniforme à la paroi.

ALLGEMEINE BESCHREIBUNG UND KORRELATION DER NATÜRLICHEN KONVEKTION UND DER MISCH-KONVEKTION IN PORÖSEN MEDIEN AUSSERHALB DES DARCY'SCHEN BEREICHS

Zusammenfassung—Durch Einführung spezieller Transformationsvariabler ergeben sich universelle Ähnlichkeitsgleichungen für die Misch-Konvektion außerhalb des Darcy'schen Bereichs entlang einer isothermen senkrechten Platte und einer gleichförmig beheizten waagerechten Platte in einem porösen Medium. Es werden sehr genaue Ähnlichkeitslösungen und Korrelationsgleichungen für die örtliche Nusselt-Zahl vorgestellt, und zwar für den gesamten Bereich der Strömungsträgheit innerhalb und außerhalb des Darcy'schen Gebietes für jede Intensität der Mischkonvektion von der Grenze der erzwungenen Strömung bis zur Grenze der natürlichen Strömung. Für eine natürliche Konvektionsströmung an einer senkrechten gleichförmig beheizten Platte wird die örtliche Nusselt-Zahl in einem großen Bereich der Strömungsträgheit mit Hilfe eines Finite-Differenzen-Verfahrens berechnet und anschließend korreliert.

УНИВЕРСАЛЬНЫЕ ФОРМУЛИРОВКИ И ОБОБЩАЮЩИЕ СООТНОШЕНИЯ ДЛЯ ЕСТЕСТВЕННОЙ И СМЕШАННОЙ КОНВЕКЦИИ В ПОРИСТЫХ СРЕДАХ, НЕ ОПИСЫВАЕМОЙ ЗАКОНОМ ДАРСИ

Аннотация—С помощью введения соответствующих переменных получены универсальные автомодельные уравнения для не описываемой законом Дарси смешанной конвекции вдоль изотермической вертикальной пластины и горизонтальной пластины с однородным тепловым потоком, которые погружены в пористую среду. Представлены точные автомодельные решения и обобщающие соотношения для локальных чисел Нуссельта, включая режимы, описываемые и не описываемые законом Дарси, когда конвекция изменяется от чисто вынужденной до чисто естественной. Приводятся также конечно-разностное решение и обобщенное соотношение для локального числа Нуссельта в условиях естественной конвекции вдоль вертикальной пластины с однородным тепловым потоком на стенке.